TEMPERATURE DEPENDENCE OF SOME SURFACE EFFECTS IN A MAGNETIC FLUID

B. M. Berkovskii and N. N. Smirnov

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The temperature dependence of the shape of the surface of a drop of magnetic fluid in a uniform external magnetic field is studied, and temperatures corresponding to the extremum values of the elongation of the drop are found.

It is known that the shape and surface area of a magnetic fluid drop strongly affects heat- and mass-transport characteristics. Therefore, in applications to heat- and mass-exchange devices it is necessary to be able to calculate the shape of the surface of the magnetic fluid at different temperatures.

We study the temperature dependence of the elongation of a drop of magnetic fluid. The temperature dependence calculated here can be used to study a wider class of surface effects.

We consider an isolated, magnetic fluid of bounded volume, placed in an external, uniform magnetic field $\vec{H}_0 || z$ in the absence of gravity. Let the drop be heated or cooled so slowly that its internal temperature at each instant of time can be considered to be constant. We study the shape of the free surface of the drop as a function of temperature.

Over a wide range of the parameters it can be assumed that inside the drop is established a uniform magnetic field [1, 2]

$$\vec{H}_1 = \frac{\vec{H}_0}{1+(\mu_1-1)n}, \quad n = \frac{1-\varepsilon^2}{2\varepsilon^3}\left(\ln\frac{1+\varepsilon}{1-\varepsilon} - 2\varepsilon\right), \quad \varepsilon = \sqrt{1-\frac{1}{e^2}}, \tag{1}$$

where $\mu_1 = 1 + M(H_1)/H_1$, and e is the elongation of the drop (the ratio of the length along the symmetry axis of the drop to its width). Then for an arbitrary dependence of the equilibrium magnetization $M = M(H_1)$ on the magnetic field, the surface of the drop can be found from the following expression [2]:

$$z = \int \frac{C_1 \operatorname{sgn}(z')|I_1(C_1 r)|\, dr}{\sqrt{\Pi^2 I_0^2(C_1 r) - C_1^2 I_1^2(C_1 r)}} + C_2, \tag{2}$$

where the integration is carried out with respect to an interval in r for which the expression under the square root is positive; I_ν is a modified Bessel function, $\Pi = \mu_0(M_b^2 - M_u^2)L/(2\sigma)$ is a dimensionless parameter, the subscripts u and b refer to the regions above and below the surface, respectively. The constants C_1 and C_2 are found from the condition that the volume V of the drop be constant. The elongation e of the magnetic fluid drop is determined by the single dimensionless parameter $\Pi = \mu_0 M^2 L/(2\sigma)$, and the function $e = f(\Pi)$ monotonically increases and can be found from (1) and (2). If we take $L = (V/2)^{1/3}$ as a characteristic length, then for $\Pi > 50$ one can put $e = (2\Pi)^{0.42}$ within an error of a few percent and $e = 0.92\sqrt{\Pi}$ for $50 \leqslant \Pi \leqslant 500$ within an error of about 10%. A numerical study of the minimum of the second variation of the potential [3] shows that in the approximation (1) the surface of the magnetic fluid drop as given by (2) is stable.

We take the Langevin form [4] for the dependence of the equilibrium magnetization M:

$$M = \varphi_f M_S \operatorname{La}(\mu_0 H V_f M_S/(k_B T)), \quad \operatorname{La}(x) = \operatorname{cth}(x) - x^{-1}, \tag{3}$$

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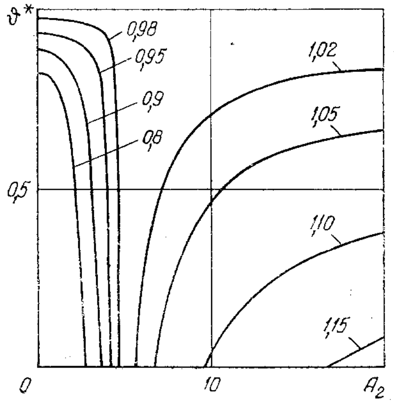

Fig. 1. Dependence of the dimensionless temperature ϑ^* for which the extremum value of the elongation occurs on the dimensionless parameters A_2 and τ. The numerals next to the curves refer to values of τ.

and the ferromagnetic saturation magnetization M_S as a function of temperature T is given by the Weiss law [5]:

$$M_S = M_0 \sqrt{3(1-T/\Theta)},\ T<\Theta, \tag{4}$$

The surface tension of the fluid at temperatures ranging from the solidifying point up to the critical temperature T_c is given by a relation of the Katayam–Gugenheim type [6]:

$$\sigma = \sigma_0 (1 - T/T_c)^k,\ k = 1.23. \tag{5}$$

The dependence of the volume V_i of the carrier fluid ($i = l$) and of the solid ferromagnetic particles ($i = f$) on temperature and pressure p is given by the empirical Tait formula [7]:

$$V_i/V_0 = (1 - n_T \beta p)^{-1/n_T},\ \beta = \beta(T),\ n_T = n_T(T), \tag{6}$$

where V_0 is equal to the volume V_i at p = 1 atm, $T = T_0$.

We first consider the temperature dependence of the elongation e = e(T) without taking into account thermal expansion, i.e., we put V = const, L = const. Then the elongation, according to (1)-(5), is determined by a set of four dimensionless parameters:

$$e = f(\Pi) = f(A_1 \Pi_1 (A_2,\ \tau,\ \vartheta)), \tag{7}$$
$$\Pi_1 = 3(1 - T/\Theta)\,\mathrm{La}^2(x)/(1 - T/T_c)^k,\ x = A_2 \sqrt{3(1 - T/\Theta)}/T,$$

where $\vartheta = T/T_{max}$ is a dimensionless temperature, $T_{max} = \min(\Theta,\ T_c)$ is the maximum value of the temperature for which the magnetic fluid has both a magnetization and surface tension $\tau = T_c/\Theta$ is the ratio of characteristic temperatures of the magnetic fluid, $A_1 = \mu_0 L \varphi_f^2 M_0^2/(2\sigma_0)$ is a dimensionless length of the magnetic fluid drop, $A_2 = \mu_0 H V_f M_0/(k_B T_{max})$ is a dimensionless magnetic field.

Numerical calculations done with the help of (1), (2), (7), show that when the temperature T is changed, the surface of the drop changes in a different way for the following three types of magnetic fluids: 1) the fluid has a high critical temperature $T_c > 1.23\,\Theta$, for example, iron or magnetic in mercury; 2) the critical temperature T_c is much higher than the Curie ferromagnetic temperature $\Theta < T_c < 1.23\,\Theta$, for example, nickel in water; 3) the most typical type of magnetic fluid in which $T_c < \Theta$, for example iron, magnetite, or cobalt in water or alcohol.

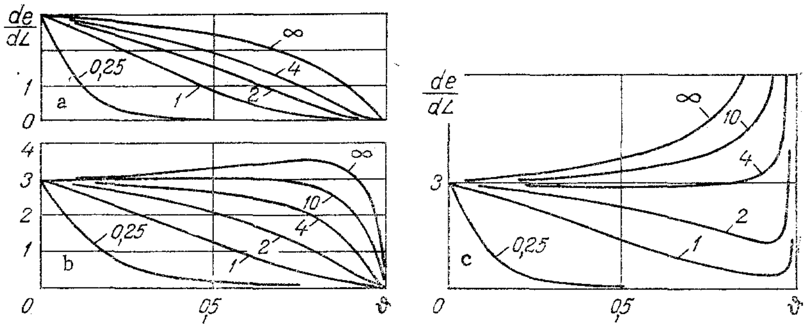

Fig. 2. Change in the elongation of a small drop of a weakly magnetic fluid, de/dL (1/m) with increase in the volume, as a function of temperature ϑ: a) $\tau = 1.5$; b) $\tau = 1.05$; c) $\tau = 1/1.05$; the numerals labelling the curves refer to the values of A_2. The parameters ϑ, τ, A_1, A_2, e are dimensionless, $V \leqslant 1$ mm^3, $M_0 = 2.82 \cdot 10^5$ A/m, $\sigma = 2.5 \cdot 10^{-2}$ N/m, $\varphi_f = 1.56 \cdot 10^{-3}$.

A drop of magnetic fluid of the first kind (high critical temperature) contract when heated (i.e., the elongation e goes down) for any magnetic field.

For a magnetic fluid of the second kind ($1 < \tau < k$) there exists a critical value of the dimensionless magnetic fluid $A_{2ma}(\tau) = 2/(\sqrt{3}(k/\tau - 1)) \geqslant 2/(\sqrt{3}(k-1)) \approx 5.02$ such that for a magnetic field above this critical value ($A_2 > A_{2ma}$), the drop at first elongates with increasing temperature and then contracts; the temperature at which the maximum value of elongation is reached $\vartheta^* = \vartheta^*(A_2)$ increases with increasing magnetic field (Fig. 1).

In very strong magnetic fields ($A_2 \to \infty$) in expression $\vartheta^* = (k - \tau)/(k - 1)$ is valid and the drop has the maximum elongation at a temperature $T^* = (k\Theta - T_c)/(k - 1)$. For a magnetic fluid of the second kind the temperature corresponding to the maximum elongation of the drop varies in strong magnetic fields within wide limits: from the solidifying temperature of the fluid ($T^* \to 0$ when $\tau \to k$) to the Curie ferromagnetic temperature ($T^* \to \Theta$ for $\tau \to 1+0$) . In a magnetic field below the critical field ($A_2 < A_{2ma}$) the drop contracts for all values of the temperature when heated.

In magnetic fluids of the third kind ($T_c < \Theta$) there exists a critical value of the magnetic field $A_{2mi}(\tau) = 2/(\sqrt{3}(k - \tau)) \leqslant 2/(\sqrt{3}(k - 1)) \approx 5.02$ such that for large magnetic fields ($A_2 > A_{2mi}$) the magnetic fluid drop elongates with heating at any value of the temperature. In weaker magnetic fields ($A_2 < A_{2mi}$) with increasing temperature the drop at first contracts (for $\vartheta < \vartheta^*$), then (for $\vartheta > \vartheta^*$) elongates. The temperature $\vartheta^*(A_2)$ at which the minimum elongation of the drop is reached decreases with increasing magnetic field (Fig. 1).

For the typical magnetic fluid (type 3), where M_S = 27 kA/m, $\rho = 1.37 \cdot 10^3$ kg/m^3, σ = 0.05 N/m at T = 20°C [4], the critical dimensionless magnetic field has the value A_{2mi} = 2.89 which for magnetite particles of volume $V_f = 5 \cdot 10^{-23}$ m^3 corresponds to a critical field H = $1.45 \cdot 10^3$ A/m. A drop of WM-27 of volume 0.1 cm^3 for this value of H increases in elongation by 9% upon heating from 20 to 100°C (Fig. 2).

We calculated the temperature dependence of the shape of a drop using (1)-(6), with account of thermal expansion as given by (6) for several typical materials. The temperature T*, corresponding to the extremum value of the elongation of the drop calculated with the effects of thermal expansion (6) included, changed by less than 20° from the T* calculated without (6) (Fig. 1). The inclusion of thermal expansion (6) in determining T* is important only in magnetic fluids with $T_c/\Theta \geqslant 2$; with the inclusion of (6) the same dependence $T^* = T^*(A_2)$ on magnetic field obtains; a magnetic fluid drop of the first kind contracts upon heating for all magnetic fields, in a magnetic fluid drop of the second kind $\vartheta^* = \vartheta^*(A_2)$ increases with increasing magnetic field, etc.

From the numerical results, we can make the following conclusions.

1. The temperature T* corresponding to the extremum value of the elongation of a magnetic fluid drop with T_c not very high ($T_c/\Theta \leqslant 2$) can be calculated without taking into account thermal expansion.

2. Magnetic fluids for which $T_c < \Theta$ (iron, cobalt, magnetite in alcohol or water) are the most sensitive to changes in temperature; the least sensitive are magnetic fluids with a high critical temperature ($T_c > 1{,}23\,\Theta$).

3. Magnetic fluids having $e = e(T)$ most sensitive to the magnetic field strength have $T_c < \Theta$.

4. The basic features of the temperature dependence of the elongation of magnetic fluid drops (the characteristic decrease and increase for the three types of fluids, the value T*, the dependence on magnetic field strength, etc.) also are observed in other surface effects in magnetic fluids which depend monotonically on a single dimensionless parameter Π.

5. The temperature T* found above (Fig. 1) defines the optimal experimental conditions for observing the elongation of an isolated volume of magnetic fluid, as well as other surface effects, and thus one can predict theoretically the optimal temperature regime for thermal and mass exchange devices.

NOTATION

Π, Π_1, A_1, A_2, C, C_1, C_2, dimensionless parameters; μ_0, vacuum magnetic permeability; M, equilibrium ferromagnetic magnetization; L, characteristic linear dimension of the magnetic fluid surface; σ, surface tension; T, temperature; φ , volumetric concentration of the ferromagnetic phase; La, Langevin function; H, magnetic field strength in the magnetic fluid; V_f, volume of ferroparticle; k_B, Boltzmann constant; Θ, Curie ferromagnetic temperature; T_c, critical temperature of the fluid; V_l, volume of the carrier fluid; p, pressure; e, elongation of the magnetic fluid drop; H_0, external magnetic field strength; μ_1, relative magnetic permeability of the magnetic fluid; z, coordinate along the axis; r, distance from the axis z; V, volume of the drop.

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